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Signatures of the Dirac electron in the flux dependence of total persistent currents in isolated Aharonov–Bohm rings

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Abstract
This paper deals with the total persistent current at $T = 0$ produced by the exact energy solution of the Dirac electron moving on isolated 1D Aharonov–Bohm rings. Leading contributions concerning the non-relativistic limit are written down for large values of the electron number. Usual non-relativistic currents get reproduced, but now in terms of a reversed parity of the electron number. Such an 'anomaly' is able to serve as a signature of the Dirac electron referred to above.

1. Introduction

Persistent currents in isolated small 1D rings threaded by a magnetic flux have attracted appreciable interest for nearly half a century [1–3]. Such currents, which concern a large number of mesoscopic systems ranging from L-ring circuits [4] to nanotubes [5], have been observed in experiments with GaAs/GaAlAs self-assembled rings [6]. Besides the influence of the ring discretization [7, 8], the role of impurities [3, 9] and of the electron–electron interaction [3, 10] has also been accounted for. The effect of the Rashba spin–orbit interaction (SOI) [11] on electrons moving on 1D rings has also received much attention in connection with the increasing role of the spin in several areas like magnetotransport through 1D rings [12], persistent currents [13], the quantum spin Hall effect [14] or nanowires under transverse electric fields [15]. On the other hand we are aware that the SOI arises as a single particle effect from the Dirac equation, which also means that further applications, such as the relativistic Hall effect [16], remain desirable. Moreover, there are actually massless Dirac fermions in 2D, namely layers of graphene, which have been isolated recently in experiments [17]. These Dirac-like electronic systems exhibit a number of unusual properties such as the unconventional quantum Hall effect [18], semimetal–insulator transitions under a transverse magnetic field [19], magnetic oscillations [18, 20] or the onset of a conductivity minimum [17]. This has led to the explanation of related 'anomalies' concerning the unusual quantization of the Hall conductivity at half-integer filling factors or the onset of an additional phase shift of π provided by Shubnikov–de Haas oscillations.

We shall use this opportunity to show that there is still a further ‘anomaly’, this time concerning the flux dependence of total persistent currents in isolated 1D Aharonov–Bohm (AB) rings without leads. Neglecting the electron–electron interaction as well as the influence of impurities, we found that the total persistent current at zero temperature ($T = 0$) produced by the exact energy of the Dirac electron on the AB ring reproduces well-known non-relativistic results [7], but with a reversed parity of the electron number. This corresponds to the appearance of an additional flux-shift of $1/2$ in the dependence of the current on the number of flux quanta, i.e. on the dimensionless flux parameter $\beta = \Phi/\Phi_0$. Here $\Phi = \pi B R^2$ stands for the magnetic flux through the 1D ring of radius R, while $\Phi_0 = hc/e$ denotes the flux quantum.

2. The energy of the Dirac electron on the circle

An electron moving on a circle with the circumference $L = 2\pi R$ is described by the stationary Dirac equation [21]

$$\mathcal{H}\Psi(\varphi) = \mathcal{E}\Psi(\varphi) \tag{1}$$

where $\Psi^{\mathrm{T}}(\varphi) = (\Phi_1, \Phi_2; \chi_1, \chi_2)$ stands for the transposed four-spinor. Resorting to cylindrical coordinates leads to the matrix Hamiltonian

$$\mathcal{H} = \begin{pmatrix} m_0c^2 & 0 & 0 & T_- \\ 0 & m_0c^2 & T_+ & 0 \\ 0 & T_- & -m_0c^2 & 0 \\ T_+ & 0 & 0 & -m_0c^2 \end{pmatrix} \tag{2}$$

where

$$T_\pm = \pm\frac{\hbar c}{R}\exp(\pm \mathrm{i}\varphi)\frac{\partial}{\partial\varphi}. \tag{3}$$

The polar angle is denoted as usual by $\varphi \in [0, 2\pi]$. The influence of a transverse magnetic field $\vec{B} = B\vec{e_z}$ is incorporated by virtue of the minimal substitution

$$\frac{\partial}{\partial\varphi} \to \frac{\partial}{\partial\varphi} + \mathrm{i}\beta \tag{4}$$

working in the symmetric gauge $\vec{A} = (\vec{B} \times \vec{x})/2$. One realizes that (1) can be solved in terms of planar waves like

$$\Psi(\varphi) \sim \exp(\mathrm{i}(r - \beta)) \tag{5}$$

provided that

$$r = \frac{1}{2}\left[1 \pm \left(1 + \frac{4R^2}{\hbar^2 c^2}(\mathcal{E}^2 - m_0^2c^4)\right)^{1/2}\right]. \tag{6}$$

In addition, the periodicity condition

$$\Psi(\varphi + 2\pi) = \Psi(\varphi) \tag{7}$$

has to be accounted for, which results in the energy solution

$$\mathcal{E} = \mathcal{E}_m(\beta) = \left[m_0^2c^4 + \frac{\hbar^2c^2}{R^2}(m + \beta)(m + \beta - 1)\right]^{1/2} \tag{8}$$

where $m = 0, \pm 1, \pm 2, \ldots$ is an arbitrary integer. Note that the term $(m + \beta)^2$ under the square root can be traced back to the Aharonov–Bohm effect [7].

Next one realizes that by virtue of the choice of $T = 0$, we have to select energies, say $E = E_m(\beta)$, which are smaller than the Fermi one, i.e. than $E_{\mathrm{F}} = \hbar^2 k_{\mathrm{F}}^2/2m_0$. This amounts to introducing the difference

$$E_m(\beta) = \mathcal{E}_m(\beta) - m_0c^2 \tag{9}$$

instead of $\mathcal{E}_m(\beta)$, which exhibits a magnitude of the orders needed.

3. Parameter fixings and related approximations

We have to realize that specifying the Fermi wavevector k_F results in a fixed value N_e of the number of electrons. This number reads

$$N_e = 2Rk_F = \frac{L}{\pi}k_F \tag{10}$$

which proceeds in accord with the free electron description, i.e. by virtue of a direct enumeration of phase-space cells. Using (9), one sees that energies smaller than E_F are provided by selected m-intervals for which

$$(m+\beta)(m+\beta-1) \leqslant \frac{N_e^2}{4}(1+\varepsilon) \tag{11}$$

where

$$\varepsilon = \frac{E_F}{2m_0c^2} = \frac{N_e^2}{16}\frac{l_C^2}{R^2} \tag{12}$$

and $l_C = \hbar/m_0c$. It is understood that the edges of such intervals, which are characterized by integer m-realizations, are of a special interest for the formulation of a closed description. One realizes that a such problem is hardly tractable, if not impossible, in terms of arbitrary values of the parameter ε. However, the parameter ε is rather small in practice. Indeed, the Fermi energy of metals and semiconductors has, in general, a magnitude of the order of some few eVs, while $m_0c^2 \approx 5\times10^5$ eV. Accordingly, $\varepsilon \approx 10^{-6}$, which confirms our expectations. On the other hand one has $\varepsilon \cong 0.9N_e^2 10^{-12}$ for a mesoscopic ring with a typical small radius of about $R = 100$ nm. This means that large N_e values like $N_e \cong 10^3$, which are characteristic for small metallic rings [22], comply with negligible $\varepsilon \approx 10^{-6}$ magnitudes just referred to above. Note that in this latter case the Fermi velocity has a magnitude of the order of 6×10^7 cm s^{-1}. Moreover, mesoscopic metallic rings for which $R \approx 1$ μm, such as found in the case of single isolated gold loops [23] or cooper rings [24], can also be invoked. This leads, of course, to yet larger values of N_e. The point is that by virtue of such data one meets the non-relativistic $\varepsilon \to 0$ limit, but useful simplifications can also be made.

So there are reasons for replacing (11) with the simplified inequality

$$(m+\beta)(m+\beta-1) \leqslant \frac{N_e^2}{4} \tag{13}$$

which differs, however, from the non-relativistic descriptions discussed before [7, 8]. Next it is clear that (13) has to be handled in terms of basic flux intervals of unit length like $\beta \in [\beta_1, \beta_2]$, which relies on the unit flux periodicity of the Fourier series description of related β-dependent currents. One has $\beta_2 - \beta_1 = 1$ and $|\beta_i| \leqslant 1$, with the understanding that β_i $(i = 1, 2)$ is sensitive to N_e. One should then have $[\beta_1, \beta_2] = [0, 1]$ or $[\beta_1, \beta_2] = [-1/2, 1/2]$, respectively.

One realizes that (13) leads, in general, to selected m-intervals like

$$m \in [-M_-(\beta), M_+(\beta)] \tag{14}$$

where

$$M_\pm(\beta) = \mathrm{Int}\left[\left(\left(\beta-\frac{1}{2}\right)^2 + \frac{N_e^2}{4}\right)^{1/2} \mp \left(\beta-\frac{1}{2}\right)\right] > 0 \tag{15}$$

are pertinent integer-part functions. However, we have to look for a closed description, i.e. for integer realizations of both $M_\pm(\beta_1)$ and $M_\pm(\beta_2)$ edges. An explicit solution can then be readily found using large N_e values discussed above. This means that the square root in (15) can be approximated reasonably as

$$\left(\left(\beta-\frac{1}{2}\right)^2 + \frac{N_e^2}{4}\right)^{1/2} \simeq \frac{N_e}{2} \tag{16}$$

so that

$$M_{\pm}(\beta) \cong \frac{N_e}{2} \mp \left(\beta - \frac{1}{2}\right). \tag{17}$$

Accordingly, one has

$$\beta \in [\beta_1, \beta_2] \tag{18}$$

where

$$\beta_i = \beta_i(N_e) = \frac{(-1)^i}{2} + \frac{1-(-1)^{N_e}}{4} \tag{19}$$

so that $\beta \in [-1/2, 1/2]$ and $\beta \in [0, 1]$ if N_e is even and odd, respectively.

4. The derivation of total currents

Now we are ready to derive total persistent currents at $T = 0$ characterizing the Dirac electron on the 1D AB ring. First, the current carried by (9) is given by

$$I_m(\beta) = -\frac{c}{\Phi_0}\frac{\partial}{\partial\beta}E_m(\beta) \tag{20}$$

so that

$$I_m(\beta) = -\frac{2}{N_e}I_F\frac{m+\beta-1/2}{\Gamma_m(\varepsilon,\beta)}. \tag{21}$$

The present current scale is

$$I_F = \frac{e\hbar k_F}{2\pi R m_0} \tag{22}$$

whereas

$$\Gamma_m(\varepsilon,\beta) = \left[1 + \frac{16\varepsilon}{N_e^2}(m+\beta)(m+\beta-1)\right]^{1/2}. \tag{23}$$

This yields the total persistent current

$$I(\beta,\varepsilon) = -\frac{2}{N_e}I_F\sum{}^{*}\frac{m+\beta-1/2}{\Gamma_m(\varepsilon,\beta)} \tag{24}$$

where the asterisk means that the summation should be done in accord with (11). So far we have just to say that $I(\beta,\varepsilon)$ is subject to an upper bound like

$$I(\beta,\varepsilon) \leqslant -\frac{2}{N_e}I_F^{*}\frac{m+\beta-1/2}{1+2\varepsilon} \tag{25}$$

in accord with (11) and (23). However, the problem remains, namely the introduction of an explicit summation interval.

However, applying the $\varepsilon \to 0$ limit yields $\Gamma_m(0,\beta) = 1$, in which case the total current is given, in general, by

$$I^{(\pm)}(\beta) = \tfrac{1}{2}[I_1^{(\pm)}(\beta) + I_2^{(\pm)}(\beta)] \tag{26}$$

where

$$I_i^{(\pm)}(\beta) = -\frac{2}{N_e}I_{F_{S_i^{(\pm)}}}\left(m+\beta-\frac{1}{2}\right). \tag{27}$$

The summation intervals displayed above look like

$$S_i^{(\pm)} = [-M_-^{(\pm)}(\beta_i), M_+^{(\pm)}(\beta_i)] \tag{28}$$

which proceeds in accord with (17) and (19). The total currents produced by the present non-relativistic limit are then given by

$$I^{(\pm)}(\beta) = I_1^{(\pm)}(\beta) = I_2^{(\pm)}(\beta) = -I_F \frac{N_e+1}{N_e}\left(2\beta - \frac{1+(-1)^{N_e}}{2}\right) \tag{29}$$

where $(N_e+1)/N_e \cong 1$ and where the superscripts are given by $\pm = (-1)^{N_e}$. Such currents reproduce the actual non-relativistic results written down before [8], but in terms of an inverted parity. Equivalently, non-relativistic results get reproduced precisely if β in (29) is replaced by the shifted expression

$$\beta \to \beta + \frac{(-1)^{N_e}}{2} \tag{30}$$

which can also be viewed as reflecting the influence of the spin. Accordingly the non-relativistic current is given by

$$I_{NR}^{(\pm)}(\beta) = -I_F \frac{N_e+1}{N_e}\left(2\beta + \frac{(-1)^{N_e}-1}{2}\right) \tag{31}$$

which works in conjunction with the shifted β-interval

$$[\beta_1, \beta_2] \to \left[\beta_1 + \frac{(-1)^{N_e}}{2}, \beta_2 + \frac{(-1)^{N_e}}{2}\right] = \left[\frac{(-1)^{N_e}-1}{4}, \frac{(-1)^{N_e}+3}{4}\right] \tag{32}$$

instead of (19). In other words, we have succeeded in establishing a further 'anomaly', able to serve as a signature of the Dirac electron.

Generalized current formulae can also be easily established with the help of Fourier series. This yields the current

$$I_{FS}(\beta, N_e) = \frac{2I_F}{\pi}\frac{N_e+1}{N_e}\sum_{l=1}^{\infty}(-1)^l \frac{\cos(Llk_F)}{l}\sin(2\pi l\beta) \tag{33}$$

which is a periodic function in β with unit period, as one might expect.

5. Conclusions

The total persistence produced by the energy of the Dirac electron on an isolated 1D AB-ring has been discussed for large values of the electron number. This opens the way to establish the non-relativistic limit, such as given by (29), which represents the main result of this paper. Equation (29) looks like the actual non-relativistic result (31), but it is different. Indeed, the current (29) originates from (8), which stands for the relativistic counterpart of the energy of the Aharonov–Bohm ring. The 'anomaly' established in this manner is also a new finding, which has its own interest. Indeed, handling the 'anomaly' amounts to changing the parity, which looks interesting from the theoretical point of view. Higher-order relativistic corrections could also be made by combining power series in ε of $1/\Gamma_m(\varepsilon, \beta)$ with (13) instead of (11). Such calculations are feasible, but they go beyond the immediate scope of this short paper. It should be mentioned that mesoscopic metallic rings with attached leads deserve further attention [12, 25].

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